Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 20, 2003*
Revised: *August 7, 2003*
Accepted: *August 22, 2003*

Black strings in asymptotically plane wave geometries

Eric G. Gimon, Akikazu Hashimoto and Oleg Lunin

Institute for Advanced Study
Princeton, NJ 08540, USA
E-mail: gimon@ias.edu aki@ias.edu, lunin@ias.edu

Veronika E. Hubeny

Department of Physics, Stanford University
Stanford, CA 94305, USA
E-mail: veronika@itp.stanford.edu

Mukund Rangamani

Department of Physics, University of California
Berkeley, CA 94720, USA, and
Theoretical Physics Group, LBNL
Berkeley, CA 94720, USA
Email: mukund@socrates.berkeley.edu

JHEP08(2003)035

Abstract: We present a class of black string spacetimes which asymptote to maximally symmetric plane wave geometries. Our construction will rely on a solution generating technique, the *null Melvin twist*, which deforms an asymptotically flat black string spacetime to an asymptotically plane wave black string spacetime while preserving the event horizon.

Keywords: Black Holes in String Theory, Penrose limit and pp-wave background.

Contents

1. Introduction

Maximally symmetric plane waves have emerged as an important background space-time of string theory [1]. Just like Minkowski, anti de Sitter, and de Sitter spaces, maximally symmetric plane wave geometries are homogeneous. Although all curvature invariants vanish, they have causal structure different from that of flat space [2]–[5] and are not globally hyperbolic [6]. The most intriguing aspect of certain maximally symmetric plane waves is the fact that they admit dual field theory description through the correspondence of [7].

Schwarzschild black holes provide important insights into the nature of gravity. While black objects in other maximally symmetric spaces are well known, analogous solutions in plane wave geometries have yet to be constructed. Nonetheless, it was shown in [8] that global null Killing isometry is consistent with the existence of an extremal event horizon. Indeed, extremal vacuum plane wave black hole solutions with regular horizon were constructed in [9]. Various extremal and non-extremal deformations of the maximally plane wave geometry were also considered in [10, 11]. More recently, pure Schwarzschild black string deformation of the six dimensional plane wave geometry with regular event horizon was constructed in [12]. Unfortunately, that Schwarzschild deformation has the effect of distorting the asymptotic geometry by a finite amount and can not be considered as a black string in an asymptotically plane wave geometry.

JHEP08(2003)035

In this article, we present a systematic solution generating technique which can be used to construct a general class of asymptotically plane wave solutions. Our method is based on the observation of [13] that certain class of plane wave geometries can be generated by applying a sequence of manipulations, which we will call the *null Melvin twist*, to Minkowski space. By applying the same sequence of manipulations starting from a black string solution, we are able to generate a large class of black string deformations of plane wave geometries with a regular horizon. These solutions are characterized by the mass density of the black string and the scale of the plane wave geometry in the rest frame of the black string. For dimensions greater than six, the deformation due to the presence of the black string decays at large distances in the directions transverse to the string. Hence in these cases, our solutions describe geometries with (radially) plane wave asymptotics. One can also construct black hole solutions in asymptotically Gödel universes [14] by further dualizing these solutions [10, 15, 16].

Since the asymptotically plane wave black string solutions we find in this paper have a regular horizon, it is straightforward to compute its area. We find that this area is identical to the area of the black string before the null Melvin twist. We will provide a proof that the null Melvin twist is a procedure which keeps the area of the horizon invariant.

The null Melvin twist can be used to generate a plane wave background with the same metric as the Penrose limit of $AdS_5 \times \mathbf{S}^5$ in type IIB supergravity which was identified in [7] as admitting a dual field theory description. Unfortunately, these two backgrounds differ from each other in their matter field content. As such, this solution generated using a null Melvin twist does not yet have an obvious dual field theory interpretation. Nonetheless, the null Melvin twist yields an explicit black string solution which should provide useful hints for finding the black string deformation of the plane wave of [7].

The organization of the paper is as follows. In section 2, we construct the black string solution in ten dimensional asymptotically plane wave background and describe some of its properties. In section 3, we describe various generalizations including adding charges and angular momenta. In section 4, we describe the analogous construction for plane wave black strings in other dimensions. Concluding remarks are presented in section 5.

2. Asymptotically plane wave black string in 10 dimensions

In this section, we describe the black string in an asymptotically 10 dimensional plane wave geometry. We will denote the maximally symmetric plane wave geometry in d dimensions by $\mathcal{P}_d$. The metric of $\mathcal{P}_{10}$ is

$$ds^2 = -dt^2 + dy^2 - \beta^2 \sum_{i=1}^{8} x_i^2\,(dt+dy)^2 + \sum_{i=1}^{8} dx_i\,dx^i\,. \tag{2.1}$$

If this metric is supported by a self dual 5-form field strength as in [1], this background is a maximally supersymmetric solution of type IIB supergravity with isometry group $\mathrm{SO}(4)\times\mathrm{SO}(4)$. In fact, there exists a one-parameter family of metrics [17, 18] with the same spacetime geometry, but with the metric supported by a combination of the RR 5-form and a RR 3-form of IIB supergravity. A generic member of this family preserves 28 supercharges and a $\mathrm{U}(2)\times\mathrm{U}(2)$ isometry group. Another special member of this one parameter family is a solution which is supported by the RR 3-form alone, with the isometry group enhanced to $\mathrm{U}(4)$. Using S-duality, we then obtain a new $\mathcal{P}_{10}$ solution which is supported entirely by fields in the NS-NS sector.

The $\mathcal{P}_{10}$ solution with NS-NS fields above is particularly well suited as a background for introducing a black string for two reasons. First, the action of the isometry group $\mathrm{U}(4)$ is transitive on the transverse seven-sphere (the orbits are $\mathrm{U}(4)/\mathrm{U}(3)$), which means that we can expect to modify the metric with functions of the transverse distance only and thus can use brute force to find the black string solution. Second, and more importantly, we can find the black string by using the fact that Minkowski space is related this $\mathcal{P}_{10}$ by a null Melvin twist.

2.1 Null Melvin twist

In this subsection, we will describe the sequence of solution generating manipulations which we call the null Melvin twist. We start by constructing a neutral black string in $\mathcal{P}_{10}$ as an example. As was shown in [13], these same manipulations applied to Minkowski space generate $\mathcal{P}_{10}$. As the construction involves only the NS-NS sector fields, they can be applied to any supergravity with an NS-NS sector. To be specific, let us describe the case where we start with a type IIB theory. To facilitate the duality transformations, we will write the metric in string frame.

1. Consider a Schwarzschild black hole solution in 8+1 dimensions [19] embedded into type IIB supergravity
$$ds^2_{\rm str} = -f(r)\,dt^2 + dy^2 + \frac{1}{f(r)}\,dr^2 + r^2\,d\Omega_7^2 \tag{2.2}$$
$$f(r) = 1 - \frac{M}{r^6} \tag{2.3}$$
which describes a black string with mass density M. This solution is translationally invariant along y.

2. Boost the geometry in the y direction by an amount γ. This gives rise to a black string with net momentum $P_y = M\sinh\gamma\cosh\gamma$.

3. T-dualize along y. This gives a solution of IIA supergravity with fundamental string charge $Q_{F1} = M\sinh\gamma\cosh\gamma$. Translation along y and SO(8) rotations along the transverse $\mathbf{S}^7$ remain isometries of this geometry.

4. Twist the rotation of $\mathbf{S}^7$ along y. By twisting, we mean parameterizing the plane transverse to the string in cartesian coordinates x_i, and making the following change of coordinates
$$\begin{aligned} x_1 + ix_2 &\to e^{i\,\alpha\,y}(x_1+ix_2)\\ x_3 + ix_4 &\to e^{i\,\alpha\,y}(x_3+ix_4)\\ x_5 + ix_6 &\to e^{i\,\alpha\,y}(x_5+ix_6)\\ x_7 + ix_8 &\to e^{i\,\alpha\,y}(x_7+ix_8)\,, \end{aligned} \tag{2.4}$$

where the parameter α controls the amount of twisting. For the sake of simplicity, we twist in all four planes of rotation by the same amount. This has the effect of replacing the metric on the 7-sphere according to

$$d\Omega_7^2 \to d\Omega_7^2 + \alpha\,\sigma\,dy + \alpha^2\,dy^2\,, \tag{2.5}$$

where

$$\frac{r^2\,\sigma}{2} = x_1 dx_2 - x_2 dx_1 + x_3 dx_4 - x_4 dx_3 + x_5 dx_6 - x_6 dx_5 + x_7 dx_8 - x_8 dx_7\,. \tag{2.6}$$

5. T-dualize along y. This geometry now corresponds to a black string in type IIB supergravity with momentum $P_y = M \sinh\gamma \cosh\gamma$ sitting at the origin of the Melvin universe [20], and has a U(4) isometry group.

6. Boost the solution by $-\gamma$ along y. The purpose of this boost is to cancel the net momentum due to the original boost performed at step 2.

7. Now, we perform a double scaling limit, wherein the boost γ is scaled to infinity and the twist α to zero keeping

$$\beta = \frac{1}{2}\,\alpha\, e^{\gamma} = \text{fixed}\,. \tag{2.7}$$

The end result is the black string in an asymptotically $\mathcal{P}_{10}$ spacetime.

$$\begin{aligned}
ds^2_{\rm str} &= -\frac{f(r)\,\left(1+\beta^2\,r^2\right)}{k(r)}\,dt^2 - \frac{2\,\beta^2\,r^2\,f(r)}{k(r)}\,dt\,dy + \left(1 - \frac{\beta^2\,r^2}{k(r)}\right)\,dy^2 + \\
&\quad + \frac{dr^2}{f(r)} + r^2\,d\Omega_7^2 - \frac{\beta^2\,r^4\,(1-f(r))}{4\,k(r)}\,\sigma^2 \\
e^{\varphi} &= \frac{1}{\sqrt{k(r)}} \\
B &= \frac{\beta\,r^2}{2k(r)}\,\left(f(r)\,dt + dy\right)\wedge\sigma\,,
\end{aligned} \tag{2.8}$$

where

$$k(r) = 1 + \frac{\beta^2 M}{r^4}\,. \tag{2.9}$$

Steps 3 through 5 are the manipulations involved in generating an ordinary Melvin flux tube solution by twisting along a spatial isometry direction y which was originally described in [21]. Steps 2 and 6 boost the direction in the t-y plane along which the Melvin twist is performed. The final step has the effect of scaling the isometry direction of the Melvin twist to be null. It is therefore natural to refer to this sequence of steps as the null Melvin twist.

JHEP08(2003)035

2.2 Properties of the asymptotically $\mathcal{P}_{10}$ black string solution

The solution (2.8) is very simple. By inspection, if we set M to zero the solution reduces to the $\mathcal{P}_{10}$ geometry described at the beginning of this section. On the other hand, setting $\beta = 0$ will reduce the solution to the black string solution (2.2). There is a regular horizon at

$$r_H = M^{1/6} \tag{2.10}$$

which persists for finite values of β. One can therefore interpret (2.8) as the black string deformation of $\mathcal{P}_{10}$. Furthermore, since both $f(r)$ and $k(r)$ asymptote to 1 as r is taken to be large, the effect of M decays at large r. Unlike the six dimensional solution described in [12] which deformed the geometry by a finite amount at large r, (2.8) is a black string solution which genuinely asymptotes to $\mathcal{P}_{10}$.

Because both the dilaton and the metric asymptote to $\mathcal{P}_{10}$ in (2.8), one can unambiguously define the area of the horizon in Einstein frame with the same asymptotics. The area of the fixed (r,t)-surface in Einstein frame for (2.8) is

$$\mathcal{A} = L\sqrt{k(r) - \beta^2 r^2}\, r^7\, \Omega_7\,, \tag{2.11}$$

where $\Omega_7 = \pi^4/6$ is volume of a unit $\mathbf{S}^7$ and L is the length of the translationally invariant y-direction. At the horizon, this evaluates to

$$\mathcal{A}_H = L\, M^{7/6}\, \Omega_7\,. \tag{2.12}$$

It is tempting to interpret this area in Planck units

$$S = \frac{L\, M^{7/6}\, \Omega_7}{4\, G_{10}} \tag{2.13}$$

as an entropy of some sort. A remarkable fact is that this quantity is independent of the parameter β.

It would also be interesting to compute the temperature associated to this black string. Ordinarily one computes the temperature of a black object in terms of the surface gravity

$$\kappa^2 = -\frac{1}{2}\left(\nabla^a \xi^b\right)(\nabla_a \xi_b)\,, \tag{2.14}$$

where ξ^a is the null generator of the horizon. The temperature is then given in terms of the surface gravity as $T_H = \kappa/2\pi$.

For the solution (2.8), the null generator of the horizon is simply the Killing vector

$$\xi^a = \left(\frac{\partial}{\partial t}\right)^a\,. \tag{2.15}$$

Special care is necessary with regards to the normalization of this Killing vector in the computation of the temperature. In asymptotically flat space, for example, it is natural to normalize the Killing vector so that it is of unit norm asymptotically. It is a priori not clear what constitutes a natural normalization of the Killing vector in an asymptotically plane wave geometry. Let us therefore use a normalization such that the Killing vector takes precisely the form (2.15) and interpret the temperature as being measured in units of inverse coordinate time t. With this caveat in mind, the temperature at the horizon of the solution (2.8) is found to be

$$T_H = \frac{3}{2\pi} M^{-1/6} \,. \tag{2.16}$$

What is remarkable about these results is the fact that both the temperature and entropy are independent of the parameter β. We will in fact show in the appendix that the area of the horizon is invariant under a null Melvin twist for a general class of spacetimes. This would appear to indicate that the parameter M has a natural interpretation as the mass density of the black string. This is a rather non-trivial statement since a proper notion of mass analogous to the ADM mass for an asymptotically plane wave geometry has not yet been defined.

The solution (2.8) is free of closed time-like curves so long as the y coordinate is decompactified at the end of chain of dualities. If the y coordinate is compact, closed time-like curves can appear just as in the case of ordinary plane-waves.

3. Generalizations

The null Melvin twist construction is extremely simple and can be applied to generate a wide variety of asymptotically plane wave geometries. In this section we will describe a few examples.

3.1 Rotating black holes

One simple generalization of (2.8) is to add angular momentum. Consider a rotating black string in type IIB supergravity [22].

$$ds^2_{\rm str} = -dt^2 + (1 - f(r)) \left(dt + \frac{\ell}{2}\sigma \right)^2 + dy^2 + \frac{1}{h(r)} dr^2 + r^2 d\Omega_7^2 \,, \tag{3.1}$$

where

$$f(r) = 1 - \frac{M}{r^6} \qquad \text{and} \qquad h(r) = 1 - \frac{M}{r^6} + \frac{M\ell^2}{r^8} \,. \tag{3.2}$$

In general, rotating black strings in 10-dimensions admit four independent angular momentum charges. We have taken all four angular momenta to equal ℓ for simplicity.

Applying the null Melvin twist procedure of the previous section leads to the following solution of type IIB supergravity

$$\begin{aligned} ds^2_{\rm str} &= -\frac{f(r) + \beta^2 r^2\, h(r)}{k(r)}\, dt^2 - \frac{2\beta^2 r^2 h(r)}{k(r)}\, dt\, dy + \left(\frac{1 - \beta^2 r^2\, h(r)}{k(r)} \right) dy^2 + \\ &\quad + \frac{dr^2}{h(r)} + r^2\, d\Omega_7^2 - \frac{M}{4\, r^2\, k(r)} \left(\beta^2 - \frac{\ell^2}{r^4} \right) \sigma^2 - \frac{M\ell}{r^6 k(r)}\, \sigma\, dt \\ e^{\varphi} &= \frac{1}{\sqrt{k(r)}} \\ B &= \frac{\beta r^2}{2k(r)} \left(h(r)\, dt \wedge \sigma + (1 + \frac{M\ell^2}{r^8})\, dy \wedge \sigma + \frac{2M\ell}{r^8}\, dt \wedge dy \right) , \end{aligned} \tag{3.3}$$

where as before

$$k(r) = 1 + \frac{M\beta^2}{r^4}\,. \tag{3.4}$$

The inner and outer horizons are located at the roots of $h(r)$. Just as in the non-rotating case, this geometry asymptotes to plane wave for large r or small M, but reduces to a rotating black string in the small β limit. The horizon area and the surface gravity are independent of β and agree with the results for a rotating black string in asymptotically flat space. The outer horizon of the $\ell \neq 0$ solution carries non-vanishing angular velocity

$$\omega_H = -\frac{2\ell}{r_H^2}\,. \tag{3.5}$$

3.2 Charged black strings

It is also easy to add charges to (2.8). Simply start with the non-extremal fundamental string solution and apply the null Melvin twist. This gives

$$\begin{aligned}
ds^2_{\rm str} &= -\frac{f(r)}{k_\delta(r)}\,dt^2 - \frac{\beta^2\,r^2}{k_\delta(r)}\left(1 - \frac{M\cosh^2\delta}{r^6}\right)(dt+dy)^2 + \frac{1}{k_\delta(r)}dy^2 + \frac{1}{f(r)}\,dr^2 + \\
&\quad + r^2\,d\Omega_7^2 + \frac{\beta\,M\,\sinh 2\delta}{2\,r^4\,k_\delta(r)}\,(dt+dy)\,\sigma - \frac{\beta^2 M}{4\,r^2\,k_\delta(r)}\,\sigma^2 \\
B &= \frac{M\,\sinh 2\delta}{2\,r^6\,k_\delta(r)}\,dt\wedge dy + \frac{\beta\,r^2}{2\;k_\delta(r)}\,(f(r)dt + dy)\wedge\sigma \\
e^{\varphi} &= \frac{1}{\sqrt{k_\delta(r)}}\,.
\end{aligned} \tag{3.6}$$

Here, we introduced

$$k_\delta(r) \equiv 1 + \frac{M\sinh^2\delta}{r^6} + \frac{M\beta^2}{r^4}\,. \tag{3.7}$$

A non-extremal D-string solution can be obtained in the same way if we perform the null Melvin twist starting from the nonextremal D-string instead of the fundamental string. The extremal limit of this solution is closely related to the solutions described in [23, 24].

3.3 General twists

So far, we have considered the case where one twists the four complex planes transverse to the black string equally. One can readily generalize this to independent twists in each of the four complex planes

$$\begin{aligned}
x_1 + ix_2 &\to e^{2i\alpha\,v_1\,y}(x_1 + ix_2) \\
x_3 + ix_4 &\to e^{2i\alpha\,v_2\,y}(x_3 + ix_4) \\
x_5 + ix_6 &\to e^{2i\alpha\,v_3\,y}(x_5 + ix_6) \\
x_7 + ix_8 &\to e^{2i\alpha\,v_4\,y}(x_7 + ix_8)\,.
\end{aligned} \tag{3.8}$$

Following the chain of dualities one obtains

$$
\begin{aligned}
ds^2_{\rm str} &= -\frac{f(r)\,\left(1+\beta^2\,r^2\,|\sigma_v|^2\right)}{k_v(r,\Omega_7)}\,dt^2 - \frac{2\,\beta^2\,r^2\,|\sigma_v|^2\,f(r)}{k_v(r,\Omega_7)}\,dt\,dy + \left(1-\frac{\beta^2\,r^2\,|\sigma_v|^2}{k_v(r,\Omega_7)}\right)\,dy^2 + \\
&\quad + \frac{dr^2}{f(r)} + r^2\,d\Omega_7^2 - \frac{\beta^2\,r^4\,(1-f(r))}{4\,k_v(r,\Omega_7)}\,\sigma_v^2 \\
e^{\varphi} &= \frac{1}{\sqrt{k_v(r,\Omega_7)}} \\
B &= \frac{\beta\,r^2}{2\,k_v(r,\Omega_7)}\,\left(f(r)\,dt+dy\right)\wedge\sigma_v
\end{aligned} \tag{3.9}
$$

with

$$
\begin{aligned}
k_v(r,\Omega_7) &= 1+\frac{\beta^2 M|\sigma_v|^2}{r^4}\,, \\
\frac{r^2\sigma_v}{2} &= v_1\,(x_1dx_2-x_2dx_1)+v_2\,(x_3dx_4-x_4dx_3)+ \\
&\quad + v_3\,(x_5dx_6-x_6dx_5)+v_4\,(x_7dx_8-x_8dx_7)\,,
\end{aligned} \tag{3.10}
$$

and

$$
|\sigma_v|^2 = \frac{1}{r^2}\left(v_1^2(x_1^2+x_2^2)+v_2^2(x_3^2+x_4^2)+v_3^2(x_5^2+x_6^2)+v_4^2(x_7^2+x_8^2)\right). \tag{3.11}
$$

Note that since the U(4) isometry is broken, $k_v(r,\Omega_7)$ can now depend non-trivially on the coordinates of $\mathbf{S}^7$. One can explicitly check, though, that the horizon area and the surface gravity remain independent of β and the v_i's.

To be completely general, one can construct a solution with 13 independent parameters M, β, P_y, Q_{F1}, Q_{D1}, ℓ_1, ℓ_2, ℓ_3, ℓ_4, v_1, v_2, v_3, and v_4. We will not write this most general solution explicitly.

4. Asymptotically plane wave black string in other dimensions

Another natural generalization of our procedure is to apply the null Melvin twist to black branes smeared in more dimensions. A black p-brane in 10 dimensions, when compactified along $p-1$ of the translationally invariant directions, will look like a black string in $d=11-p$ dimensions. The $p-1$ extra dimensions play a spectator role, and so by applying the null Melvin twist on the effective d dimensional black string, one can construct black string solutions which are asymptotically $\mathcal{P}_d$.

The metric of black $(11-d)$-brane is simply

$$
\begin{aligned}
ds^2_{\rm str} &= -f_d(r)\,dt^2+dy^2+\frac{1}{f_d(r)}\,dr^2+r^2\,d\Omega_{d-3}+\sum_{i=1}^{10-d}dz_i^2\,, \\
f_d(r) &= 1-\frac{M}{r^{d-4}}\,.
\end{aligned} \tag{4.1}
$$

To simplify the discussion, let us restrict to even values of d. Then there are $(d-2)/2$ independent null Melvin twist parameters that one can independently adjust. Let us further take all the $(d-2)/2$ twist parameters to be equal for simplicity. Then, we find that the supergravity solution for the neutral black string in $\mathcal{P}_d$ takes the form

$$\begin{aligned} ds^2_{\rm str} &= -\frac{f_d(r)\left(1+\beta^2 r^2\right)}{k_d(r)}\, dt^2 - \frac{2\,\beta^2\, r^2\, f_d(r)}{k_d(r)}\, dt\, dy + \left(1-\frac{\beta^2\, r^2}{k_d(r)}\right) dy^2 + \\ &\quad + \frac{dr^2}{f_d(r)} + r^2\, d\Omega^2_{d-3} - \frac{r^4\, \beta^2\,(1-f_d(r))}{4\, k_d(r)}\, \sigma_d^2 + \sum_{i=1}^{10-d} dz_i^2 \\ e^{\varphi} &= \frac{1}{\sqrt{k_d(r)}} \\ B &= \frac{\beta\, r^2}{2\, k_d(r)}\, (f_d(r)\, dt + dy)\wedge \sigma_d \end{aligned} \tag{4.2}$$

with

$$k_d(r) = 1 + \frac{\beta^2\, M}{r^{d-6}}\,. \tag{4.3}$$

For $d>6$, (4.2) asymptotes to $\mathcal{P}_d$ and closely resembles $\mathcal{P}_{10}$ in many ways.

The $d=6$, is special in that $k_6(r)$ does not asymptote to 1. This is similar to conical deficits which arise as a result of mass deformation in 2+1 dimensions and cause the background to be deformed by a finite amount even for large r. Nonetheless, (4.2) for $d=6$ can be considered as a black string deformation of $\mathcal{P}_6$ in the sense that in the small M limit, the solution reduces to $\mathcal{P}_6$. The black string deformation of $\mathcal{P}_6$ was also constructed in [12], but was presented in a slightly different form. Let us compare our solution to the solution presented in [12] in some more detail.

For $d=6$, (4.2) becomes

$$\begin{aligned} ds^2_{\rm str} &= -\frac{(1-M/r^2)(1+\beta^2\, r^2)}{1+\beta^2 M}\, dt^2 - \frac{2\beta^2 r^2 (1-M/r^2)}{1+\beta^2 M}\, dt\, dy + \left(1-\frac{\beta^2 r^2}{1+\beta^2 M}\right) dy^2 + \\ &\quad + \frac{dr^2}{1-M/r^2} + r^2\, d\Omega_3^2 - \frac{r^2}{4}\left(\frac{\beta^2 M}{1+\beta^2 M}\right)\sigma_6^2 \\ e^{\varphi} &= \frac{1}{\sqrt{1+\beta^2 M}} \\ B &= \frac{\beta\, r^2}{2\,(1+\beta^2 M)}\left(\left(1-\frac{M}{r^2}\right) dt + dy\right)\wedge\sigma_6\,. \end{aligned} \tag{4.4}$$

The solution presented in [12, equation (26)] reads

$$\begin{aligned} ds^2 &= \left(1-\frac{2m}{r^2}\right) d\tilde{t}^2 + d\tilde{y}^2 - 2jr^2\sigma_6(d\tilde{t}-d\tilde{y}) - 2j^2 m\, r^2 \sigma_6^2 + \\ &\quad + \left(1-\frac{2m(1-8j^2 m)}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2\,. \end{aligned} \tag{4.5}$$

Here, we relabeled the coordinates (t,y) of [12] by $(\tilde{t},\tilde{y})$ to distinguish from the coordinates used in (4.4).

To facilitate the comparison one must rewrite the metric (4.4) in the coordinate chart in which (4.5) is written. This is achieved by first twisting

$$\sigma_6 \;\to\; \sigma_6 + 2\beta(dt + dy)\,, \tag{4.6}$$

and then performing a change of variables

$$\begin{aligned} t &= (1+\beta^2 M)\,\tilde{t} \\ y &= -\beta^2 M\tilde{t} - \tilde{y} \end{aligned} \tag{4.7}$$

so that the metric becomes

$$\begin{aligned} ds^2_{\rm str} = &-\left(1 - \frac{M(1+\beta^2 M)}{r^2}\right) d\tilde{t}^2 + d\tilde{y}^2 + \left(\frac{\beta}{1+\beta^2 M}\right) r^2\,(d\tilde{t} - d\tilde{y})\,\sigma_6 + \\ &+\frac{dr^2}{1-\frac{M}{r^2}} + r^2\,d\Omega_3^2 - \frac{r^2}{4}\left(\frac{\beta^2 M}{1+\beta^2 M}\right)\sigma_6^2\,. \end{aligned} \tag{4.8}$$

Now, (4.8) and (4.5) have the same form, and they can be seen to be identical if we identify the parameters according to

$$\begin{aligned} 2m &= M(1+\beta^2 M) \\ 2j &= \frac{\beta}{(1+\beta^2 M)}\,. \end{aligned} \tag{4.9}$$

So the dimensionless combination $8j^2m$ is related to the dimensionless combination $\beta^2 M$ by

$$8j^2 m = \frac{\beta^2 M}{1+\beta^2 M}\,. \tag{4.10}$$

The critical value $8j^2m = 1$, which causes the solution (4.5) to degenerate, corresponds to $\beta^2 M$ being infinite.

The fact that (4.4) do not asymptote to plane wave geometry makes the identification of quantities such as mass, entropy, and temperature even more subtle. One can nonetheless map (4.4) to Einstein frame and compute the horizon area and the surface gravity using the killing vector (2.15) for these coordinates. This yields a β independent answer

$$\mathcal{A}_H = LM^{3/2}\Omega_3 V_{T^4}\,, \qquad \kappa^2 = \frac{1}{M}\,. \tag{4.11}$$

This was essentially guaranteed based on the arguments presented in the appendix, and suggests that the parameter M can be interpreted as a physical mass.

5. Discussion

In this article, we presented a powerful technique for generating a large class of asymptotically plane wave geometries. Using this technique, we have succeeded in constructing a supergravity solution which contains an event horizon while asymptoting to the plane wave geometry.

JHEP08(2003)035

With explicit solutions at hand, one can explore various thermodynamic interpretations of the black string geometry. We find that the null Melvin twist leaves the area of the event horizon invariant, suggesting that the entropy of the black string in the plane wave geometry is identical to the entropy of the black string before taking the null Melvin twist. Furthermore, with certain assumptions regarding the definition of temperature in asymptotic plane wave geometries, we found that it too is left unchanged under the null Melvin twist. This suggests that the mass parameter of the black string solution corresponds to a physical measure of mass in some canonical way. It would be interesting to make these ideas more precise by formulating a satisfactory definition of mass and temperature in an asymptotically plane wave geometry.

We also constructed a black string solution in an asymptotically six dimensional plane wave background using the null Melvin twist technique. We showed that this solution is equivalent to the solution presented in [12], provided that we map the parameters (m, j) characterizing the mass and the characteristic scale of the plane wave in [12] to (M, β) using (4.9). For fixed j, area of the horizon is not monotonic in m, whereas for fixed β, the area is monotonic in M. It is therefore more natural to define the microcanonical ensemble as fixing β instead of fixing j.

Clearly, this null Melvin twist can be used to generate a more general class of solutions than the ones considered in this article. Unfortunately, this technique can only be used to construct certain subset of asymptotically plane wave geometries. A plane wave geometry of particular interest which can not be constructed using this technique is the one which arises as a Penrose limit of $AdS_5 \times \mathbf{S}^5$. It would be extremely interesting to find the analogous black string solution in this asymptotic geometry.

With very little effort, one can dualize the Schwarzschild black string solution in $\mathcal{P}_d$ to a black hole solution in $(d-1)$-dimensional Gödel universe $\mathcal{G}_{d-1}$. The horizon and the velocity of light surface meet only when $\beta^2 M$ is infinite.

So far, we have only considered black string solutions. It would be extremely interesting to find the solution corresponding to black holes. Unfortunately, the null Melvin twist technique involves T-duality, and can not be applied at the level of supergravity to backgrounds which do not contain at least one translational isometry direction. However, since certain plane wave backgrounds can be constructed using only fields in the NS-NS sector, it may be possible to extract such a geometry from the consideration of the sigma model for the string world sheet in this background. A related discussion in the context of duality between Kaluza-Klein monopole and the NS5-brane can be found in [25].

Acknowledgments

We would like to thank D. Berenstein, E. Boyda, S. Ganguli, O. Ganor, P. Hořava, G. Horowitz, and S. Ross for discussions. The work of E.G.G. is supported by NSF grant PHY-0070928 and by Frank and Peggy Taplin. A.H. is supported in part by DOE grant DE-FG02-90ER40542 and the Marvin L. Goldberger Fund. V.E.H. is supported by NSF

Grant PHY-9870115. O.L. is supported by NSF grant PHY-0070928. M.R. acknowledges support from the Berkeley Center for Theoretical Physics and also partial support from the DOE grant DE-AC03-76SF00098 and the NSF grant PHY-0098840.

A. Null Melvin twist and the area of the horizon

In this appendix, we will show that the area of the horizon does not change under rather general set of manipulations for which the null Melvin twist is a special case. The proof will be based on an assumption that the $B_{\mu\nu}$ field for the initial configuration is regular at the horizon.

Consider starting from a space-time with a horizon and a space-like translation symmetry along a coordinate y in type II supergravity theory. We will assume that the metric[1] is written in the Boyer-Lindquist coordinates so that there is a coordinate r which does not mix with other coordinates, allowing one to write the metric in the form

$$g_{\mu\nu} = \left(\begin{array}{c|c} g_{rr} & 0 \\ \hline 0 & M_{\mu\nu} \end{array} \right) . \tag{A.1}$$

One of the coordinates of $M_{\mu\nu}$ will be t. By horizon, we will mean a fixed (t, r) surface where g_{rr} diverges and $\det M$ vanishes. Let $A^{\mu\nu}$ denote the cofactor of $M_{\mu\nu}$. The coordinates are regular away from the horizon, then one can write

$$A^{\mu\nu} = g^{\mu\nu} \det M \,. \tag{A.2}$$

The area of the horizon is the square root of A^{tt} evaluated at the horizon.

Consider starting from a generic space-time with the properties described above, and applying the following sequence of manipulations:

1. Boost the space-time along y by γ

2. T-dualize along the y coordinate

3. Twist by making a change of coordinates

$$x^\mu \to x^\mu + \alpha^\mu y \tag{A.3}$$

4. T-dualize along y

5. Boost along y by $-\gamma$.

These manipulations give rise to a new geometry again in Boyer-Lindquist coordinates, whose horizon area is given by the square root of

$$A^{tt} + 2\sinh^2\gamma\, \alpha^\mu B_{\mu\nu} A^{y\nu} + \sinh^2\gamma\, \alpha^\mu \alpha^\nu (B_{\mu\rho} B_{\nu\sigma} + g_{\mu\rho} g_{\nu\sigma}) A^{\rho\sigma} \,. \tag{A.4}$$

[1]In this appendix, we always write the metric in the Einstein frame.

JHEP08(2003)035

Using the relation (A.2) away from the horizon, we can rewrite the last term in this expression as

$$\sinh^2\gamma\,\alpha^\mu\alpha^\nu(B_{\mu\rho}B_{\nu\sigma}+g_{\mu\rho}g_{\nu\sigma})A^{\rho\sigma}=\sinh^2\gamma\;\det M\,\alpha^\mu\alpha^\nu(B_{\mu\rho}B_\nu{}^\rho+g_{\mu\nu})\,. \tag{A.5}$$

This is a scalar multiplied by $\det M$, so it has to vanish at the horizon. The second term is an expression proportional to

$$V_\mu A^{\mu\nu}=V^\nu\det M \tag{A.6}$$

if we set $V_\mu=\alpha^\nu B_{\mu\nu}$ which we assume to be regular at the horizon. To prove the invariance of the area, it is sufficient to prove that for any regular vector V_μ, (A.6) goes to zero as we approach the horizon.

To show this we introduce a vielbein $e_\mu{}^a$ for the $D-1$ dimensional "metric" $M_{\mu\nu}$:

$$M_{\mu\nu}=\eta_{ab}e_\mu{}^a e_\nu{}^b\,. \tag{A.7}$$

Since there is no curvature singularity at the horizon, all components of the vector V^a must be regular there. In terms of the vielbein $e_\mu{}^a$ and its cofactor $E^\mu{}_a$, we can write

$$V_\mu A^{\mu\nu}=-e^\nu{}_a V^a(\det\,e)^2=-E^\nu{}_a V^a\det\,e\,. \tag{A.8}$$

Since all elements of $M_{\mu\nu}$ are finite at the horizon, we can choose all $e_\nu{}^a$ to be finite as well. This in turn leads to finite minors $E^\nu{}_a$. On the other hand, since $\det M$ vanishes at the horizon, so does $\det\,e$, proving that $V_\mu A^{\mu\nu}$ vanishes at the horizon for any value of ν.

This shows that the area of the horizon is invariant with respect to sequence of manipulations described in this appendix. In particular, by taking α^μ to rotate the four independent transverse rotations of $\mathbf{S}^7$ and scaling the twist with respect to the boost, we show that the area of the horizon is invariant under null Melvin twist.

References

[1] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of iib superstring theory*, *J. High Energy Phys.* **01** (2002) 047 [hep-th/0110242].

[2] D. Berenstein and H. Nastase, *On lightcone string field theory from super Yang-Mills and holography*, hep-th/0205048.

[3] D. Marolf and S.F. Ross, *Plane waves: to infinity and beyond!*, *Class. and Quant. Grav.* **19** (2002) 6289 [hep-th/0208197].

[4] V.E. Hubeny and M. Rangamani, *Causal structures of PP-waves*, *J. High Energy Phys.* **12** (2002) 043 [hep-th/0211195].

[5] D. Marolf and S.F. Ross, *Plane waves and spacelike infinity*, hep-th/0303044.

[6] R. Penrose, *A remarkable property of plane waves in general relativity*, *Rev. Mod. Phys.* **37** (1965) 215.

JHEP08(2003)035

[7] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[8] V.E. Hubeny and M. Rangamani, *No horizons in PP-waves*, *J. High Energy Phys.* **11** (2002) 021 [hep-th/0210234].

[9] V.E. Hubeny and M. Rangamani, *Generating asymptotically plane wave spacetimes*, *J. High Energy Phys.* **01** (2003) 031 [hep-th/0211206].

[10] C.A.R. Herdeiro, *Spinning deformations of the D1-D5 system and a geometric resolution of closed timelike curves*, hep-th/0212002.

[11] J.T. Liu, L.A. Pando Zayas and D. Vaman, *On horizons and plane waves*, hep-th/0301187.

[12] E.G. Gimon and A. Hashimoto, *Black holes in goedel universes and PP-waves*, *Phys. Rev. Lett.* **91** (2003) 021601 [hep-th/0304181].

[13] M. Alishahiha and O.J. Ganor, *Twisted backgrounds, PP-waves and nonlocal field theories*, *J. High Energy Phys.* **03** (2003) 006 [hep-th/0301080].

[14] J.P. Gauntlett, J.B. Gutowski, C.M. Hull, S. Pakis and H.S. Reall, *All supersymmetric solutions of minimal supergravity in five dimensions*, hep-th/0209114.

[15] E.K. Boyda, S. Ganguli, P. Hořava and U. Varadarajan, *Holographic protection of chronology in universes of the goedel type*, *Phys. Rev.* **D 67** (2003) 106003 [hep-th/0212087].

[16] T. Harmark and T. Takayanagi, *Supersymmetric goedel universes in string theory*, *Nucl. Phys.* **B 662** (2003) 3 [hep-th/0301206].

[17] I. Bena and R. Roiban, *Supergravity PP-wave solutions with 28 and 24 supercharges*, *Phys. Rev.* **D 67** (2003) 125014 [hep-th/0206195].

[18] J. Michelson, *A PP-wave with 26 supercharges*, *Class. and Quant. Grav.* **19** (2002) 5935 [hep-th/0206204].

[19] F.R. Tangherlini, *Schwartzschild field in N dimensions and the dimensionality of space problem*, *Nuovo Cim.* **27** (1963) 636.

[20] M.A. Melvin, *Pure magnetic and electric geons*, *Phys. Lett.* **A 8** (1964) 65.

[21] F. Dowker, J.P. Gauntlett, D.A. Kastor and J. Traschen, *Pair creation of dilaton black holes*, *Phys. Rev.* **D 49** (1994) 2909 [hep-th/9309075].

[22] R.C. Myers and M.J. Perry, *Black holes in higher dimensional space-times*, *Ann. Phys. (NY)* **172** (1986) 304.

[23] P. Bain, P. Meessen and M. Zamaklar, *Supergravity solutions for D-branes in HPP-wave backgrounds*, *Class. and Quant. Grav.* **20** (2003) 913 [hep-th/0205106].

[24] M. Cvetič, H. Lu, C.N. Pope and K.S. Stelle, *Linearly-realised worldsheet supersymmetry in PP-wave background*, *Nucl. Phys.* **B 662** (2003) 89 [hep-th/0209193].

[25] D. Tong, *NS5-branes, T-duality and worldsheet instantons*, *J. High Energy Phys.* **07** (2002) 013 [hep-th/0204186].

JHEP08(2003)035